Power and Sample size Estimation for Bioequivalence Studies

Short cursory excerpt

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The used mathematical and statistical apparatus is here only formulated for the evaluation of the pharmacokinetic metrics which are assumed log-normal distributed.
The description follows closely Diletti, Hauschke and Steinijans (1991).

For the formulas using untransformed PK metrics refer to Phillips (1990).

The TOST procedure

Let μ_T and μ_R the expected mean values of the pharmacokinetic metric (f.i. AUC, or Cmax) of the Test and Reference formulation to be compared within a bioequivalence study.
Let the interval (Θ_1, Θ_2) denote the bioequivalence acceptance range where
$0 < \Theta_1 < 1 < \Theta_2$.
Most regulatory guidances set (Θ_1, Θ_2) = (0.8, 1.25) for log-normal distributed pharmaco-kinetic metrics (i.e. AUC, Cmax). Other values may be used (f.i. (0.75, 1.3333) for widened Cmax, or (0.9, 1.1111) for NTI drugs).

The bioequivalence test problem based on the ratio μ_T/μ_R is stated as:

$H_0: \mu_T/\mu_R \leq \Theta_1 \; or \; \mu_T/\mu_R \geq \Theta_2$ (null: bioinequivalence)

$H_1: \Theta_1 < \mu_T/\mu_R < \Theta_2$ (alternative: bioequivalence)

In case of log-normal distributed pharmaco-kinetic metrics the test problem is transformed accordingly to

$H_0: \log(\mu_T/\mu_R) \leq \log(\Theta_1) \; or \; \log(\mu_T/\mu_R) \geq \log(\Theta_2)$ (bioinequivalence)

$H_1: \log(\Theta_1) < \log(\mu_T/\mu_R) < \log(\Theta_2)$ (bioequivalence)

were $log(x)$ denotes the natural logarithm. $log(\mu_T/\mu_R) = log(\mu_T) - log(\mu_R)$ is estimated by the difference of the arithmetic means of the log-transformed observations $\bar{X}_T - \bar{X}_R$.

H_0 is rejected in favor of bioequivalence if the classical (1-2α)100% confidence interval for μ_T/μ_R is included in the bioequivalence range (Westlake 1981, 1988).
The inclusion of the (1-2α)*100% confidence interval in the acceptance range is equivalent to the two one-sided t-tests (Schuirmann 1987).

Bioequivalence is in case of data from a 2x2 cross-over study concluded if the following two conditions hold true:

$$t_1 = \frac{\bar{X}_T - \bar{X}_R - \log(\Theta_1)}{s_e\sqrt{2/n}} \geq t(1-\alpha, n-2)$$

$$t_2 = \frac{\bar{X}_T - \bar{X}_R - \log(\Theta_2)}{s_e\sqrt{2/n}} \leq -t(1-\alpha, n-2)$$

where s_e is estimated from the mean squared error of an appropriate analysis of variance. $t(1-\alpha, n-2)$ denotes the (1- α)-quantile of the central t-distribution with n-2 degrees of freedom. n is the number of subjects under study.
Here we assume that the number of subjects within the two sequence groups TR and RT, respectively, are the same. This assumption is also applied for the other designs covered within the package *PowerTOST*.

Power of the TOST procedure

The power of a statistical test is the probability that the hypothesis H_0, in our case bioinequivalence, is rejected if the alternative hypothesis H_1, here bioequivalence, is true. In other words the probability of correctly accepting bioequivalence is the power of the test. The power of the two one-sided t-tests (TOST) is thus given by

$$Power = Prob(t_1 \geq t_{(1-\alpha,n-2)}\, and\, t_2 \leq -t_{(1-\alpha,n-2)}\,|bioequivalence\ holds) \quad \textbf{(I)}$$

The t_1 and t_2 values are the t-test statistics of the two one-sided t-tests described above.

Owen (1965) has showen that the pair (t_1, t_2) has a special bivariate non-central *t*-distribution and that the power based on that distribution can be calculated as the difference of two definite integrals (Owen's Q function):

$$Power = 1 - \beta = Q_{df}\left(-t_{(1-\alpha,df)}, \delta_2; 0, R\right) - Q_{df}\left(t_{(1-\alpha,df)}, \delta_1; 0, R\right) \quad \textbf{(II)}$$

where $t_{(1-\alpha,df)}$ is the (1- α) quantile of a *t*-distribution with df degrees of freedom.
df is (n-2) in case of a classical 2x2 cross-over design and

$$\Theta_0 = null\ (\text{'}true\text{'})\ ratio \quad \textbf{(IIa)}$$

$$\delta_1 = \frac{\log(\Theta_0) - \log(\Theta_1)}{s_e\sqrt{2/n}}$$

$$\delta_2 = \frac{\log(\Theta_0) - \log(\Theta_2)}{s_e\sqrt{2/n}}$$

$$R = \frac{\sqrt{df}(\delta_1 - \delta_2)}{2 \cdot t_{(1-\alpha,df)}}$$

for log-transformed pharmaco-kinetic metrics, where s_e is the residual standard error, Θ_1 and Θ_2 are the lower and upper bioequivalence acceptance bounds (usually 0.8 and 1.25).

The residual variance (s_e^2) is connected to the within-subject coefficient of variation CV by

$$s_e^2 = mse = \log(CV^2 + 1)$$

$$CV = \sqrt{\exp(s_e^2) - 1}$$

Owen's Q function is defined as:

$$Q_\nu(t, \delta; a, b) = \frac{\sqrt{2\pi}}{\Gamma\left(\frac{\nu}{2}\right) \cdot 2^{\frac{(\nu-2)}{2}}} \int_a^b \Phi\left(\frac{t \cdot x}{\sqrt{\nu}} - \delta\right) \cdot x^{\nu-1} \cdot \varphi(x) \cdot dx \qquad \textbf{(III)}$$

where $\Gamma(x)$ is the gamma-function, $\varphi(x)$ and $\Phi(X)$ are the density and cumulative distribution function of the standard normal distribution, respectively.

Owen's Q function was long part of the SAS system (SAS® Analyst 1999), but undocumented until SAS9.2. It was implemented here in the R package *PowerTOST* via numerical evaluation of the definite integral using the `integrate()` function of the package *stats*, part of the base R-project installation (see implementation details below).

Equation (**II**) can be approximated by the univariate non-central t-distribution via

$$Power \approx pt\left(-t_{(1-\alpha,n-2)}, n-2, \delta_2\right) - pt(t_{(1-\alpha,n-2)}, n-2, \delta_1) \qquad \textbf{(IV)}$$

where $pt(t, df, \delta)$ is the distribution function of the non-central t distribution with df degrees of freedom and noncentrality parameter δ.
Equation (**IV**) can further approximated, if the non-central t-distribution is approximated by a "shifted" central t-distribution, according to

$$Power \approx pt\left(-\delta_2 - t_{(1-\alpha,n-2)}, n-2\right) - pt(t_{(1-\alpha,n-2)} - \delta_1, n-2) \qquad \textbf{(V)}$$

where $pt(t, df)$ is the distribution function of the central t-distribution with df degrees of freedom.

Both approximations perform well if the degrees of freedom df are reasonable high and the obtained power is in the usually interesting range (≥ 60-70%).

Equation (**IV**) is used throughout the book from S.A. Julious (2010), without indicating that it is an approximation; and in many other papers also.

Equation (**V**) is used in the book by Chow and Liu (2009) in chapter 9 concerning sample size calculations for higher-order (replicate) crossover designs, also without indicating the approximate nature. It is also implemented in the commercial sample size software PASS 2008 (Hintze J. 2008), module "Equivalence of means/Two means in a higher order cross-over design".

Other study designs

The formulas for other study designs used in bioequivalence studies differ from the given ones only by

- the degrees of freedom df and
- the factor 2 under the square root in the denominator of the "non-centrality" parameters δ_1 and δ_2.

The factor 2 has to be replaced by the so-called design constant b_k.
This holds if the same assumptions are made as in the 2x2 cross-over, namely the number of subjects in the sequence groups or the two groups in the parallel group design are equal, the within-subject variabilities or the variabilities in the two parallel groups of the Test and Reference formulations are assumed equal and no subject by formulation interaction is incorporated in the ANOVA for replicate cross-over designs.

See the function `known.designs()` for the values of df=degrees of freedom and b_k implemented.

For the cross-over designs n is the total number of subjects and the CV to be used here is the within-subject CV (CV of the residual error).
For the two-group parallel design the sample size is, beginning with version 0.9-0, also the total number of subjects. The CV to be used here is the CV of the total variability.

Robust degrees of freedom

Beside the use of the degrees of freedom from the corresponding ANOVA model there is in PowerTOST the possibility to use the degrees of freedom according to the so-called 'robust' evaluation (aka Senn's basic estimator, see Senn (2002) and Jones&Kenward (2006)).
These df are calculated as `n-seq`.
They arose if the evaluation is done via appropriate intra-subject contrasts to estimate T-R of the (log-transformed) PK metric under analysis.

These degrees of freedom are often more appropriate if the variability (CV) was taken from a real mixed model evaluation (f.i. FDA code for ABE in replicate cross-over studies).
See the function `known.designs()` for the values of df2 = 'robust' degrees of freedom implemented.

Unbalanced (sequence) groups

The formulas (**IIa**) given above rely on the assumption of balanced (sequence) groups, i.e. equal numbers of subjects in the sequence groups of cross-over studies or equal numbers of subjects in the two groups of a parallel group design.

To allow the power calculations for unbalanced studies, common due to dropouts, the formulas for the delta's have to be modified to

(IIb)

$$\delta_1 = \frac{log(\Theta_0) - log(\Theta_1)}{s_e\sqrt{bk_{ni}\sum 1/n_i}}$$

$$\delta_2 = \frac{\log(\Theta_0) - \log(\Theta_2)}{s_e\sqrt{bk_{ni}\sum 1/n_i}}$$

In the degrees of freedom *n* has to be replaced by $\sum n_i$. The design constants bk_{ni} also change their value compared to b_k. See later on under known.designs().

This is implemented in the function `power.TOST()` of package *PowerTOST* regardless if study is balanced or unbalanced. In case of balanced design the total n is partitioned to equal number of subjects in (sequence) groups.

The sample size estimation is nevertheless done with balanced (sequence) groups using formula (IIa).

Sample size estimation

Equation **(II)** or the approximations (IV) and (V), respectively, are implicit in n – the sample size – and can be solved for given n, alpha, power to achieve, bioequivalence margins and the assumed null ('true') ratio.
The algorithm starts with a suitable chosen value of the sample size, calculates the power for that and increases / decreases this start value in steps of the number of sequence groups in the study design until the power reaches or exceed the desired level.

The start value is chosen via the large sample approximation of the power equation (Julious 2010)

$$n_0 = 0.5 b_k \left(\frac{s_e \sqrt{2} \left(z_{(1-\alpha)} + z_{(1-\beta)} \right)}{(\log(\Theta_0) - \log(\Theta_1))} \right)^2$$

if $\log(\Theta_0) < 0$, else change the difference in denominator to $\log(\Theta_0) - \log(\Theta_2)$.

in case of the 2x2 cross-over design and log-transformed data, where z_p is the p quantile of the standard normal distribution. 1-β is the power.
If Θ_0 = 1 then $z_{(1-\beta)}$ has to be replaced by $z_{(1-\beta/2)}$.
In case of Θ_0 near 1 the quantile is changed to $z_{(1-f)}$ with f=0.5*exp(-7.06*log(Θ_0)/log(Θ_2))* β according to Zhang (2003), i.e. gradually changed from $z_{(1-\beta/2)}$ to $z_{(1-\beta)}$.
b_k is again the so-called design constant, which is =2 in case of a 2x2 cross-over.

Implementation details

Owen's Q function is since V1.4.3 not implemented by direct use of equation III above but by using the relationship

$$Q_\nu(t,\delta;a,b) = pt(t, df=\nu, nct=\delta) - \frac{\sqrt{2\pi}}{\Gamma\left(\frac{\nu}{2}\right)\cdot 2^{\frac{(\nu-2)}{2}}} \int_b^\infty \Phi\left(\frac{t\cdot x}{\sqrt{\nu}} - \delta\right) \cdot x^{\nu-1} \cdot \varphi(x) \cdot dx \qquad \textbf{(IV)}$$

where pt() is the distribution function of the non-central t distribution (Chou 1992).
The distinct integral from b to ∞ is evaluated via the `integrate()` function of the R package *stats* which performs numerical integration via an adaptive algorithm.
For reasons of numerical stability a transformation of the variable x according to x=b+y/(1-y) is used which transforms the integration limits to 0 and 1.

The function to integrate over is hidden in the internal function

```
.Q.integrand(x,nu,t,delta)
```

To avoid numerical overflow in the factor before the definite integral it is calculated logarithmically within that function as

```
lnQconst <- -((nu/2.0)-1.0)*log(2.0) - lgamma(nu/2.)
```

where `lgamma(x)` is the $log(\Gamma(x))$ function from the R package *stats*.
The factor $\sqrt{2\pi}$ vanishes if the density function $\varphi(x)$ of the standard normal distribution in equation (**III**) is replaced by `exp(-0.5*x^2)`.

If abs(delta)>37.62 the distribution function of non-central t in package stats is evaluated via a crude approximation which can be rather poor for small nu. See `help(pt)`. Then `OwensQOwen()` is used, but for speed reasons only for small nu. What is small is more or less arbitrary. The implementation uses nu<29. `OwensQOwen()` is an implementation of the algorithmn 'repeated integration by parts' as described in Owen's original paper (Owen, 1965).
In case of b = ∞ and high delta the original definition for Owens Q according to equation III is used , i.e. evaluation of the distinct integral via `integrate()`.
In case of b = ∞ and low to moderate delta Owen's Q is evaluated by $pt(t, df=\nu, nct=\delta)$.

Since for really large values of nu and the upper integration limit R the integrand is a function which is zero over nearly all its range, the `integrate()` function may fail (see `help(integrate)`).
Therefore for nu>=5000 the power is calculated via the approximation using the non-central t-distribution (see below).

For an alternative implementation of the power calculation according to equation (**II**) see the function `power.equivalence.md()` of the package *MBESS*. Author of that function is Kem F. Philipps.

The **exact power** according to equation (II) is implemented in the hidden internal function
`.power.TOST(alpha=0.05, ltheta1, ltheta2, diffm, sem, df)`
where `sem` in calling this internal function has to be set to

`sem = se*sqrt(bk/n)` if study is balanced or

`sem = se*sqrt(bkni*sum(1/n)` if study is unbalanced.

This function is used by the high level functions `power.TOST()` or `sampleN.TOST()` if you set `method="exact"` (the default).

The **approximate power** according to the non-central t-distribution is implemented in the hidden internal function
`.approx.power.TOST(alpha=0.05, ltheta1, ltheta2, diffm, sem, df)`
This function is used if you set `method="noncentral"` or `method="nct"` in `power.TOST()` or `sampleN.TOST()`.

The approximation according to equation (**V**), via "shifted" central t-distribution is implemented in the hidden function
`.approx2.power.TOST(alpha=0.05, ltheta1, ltheta2, diffm, sem, df).`
This function is used if your set `method="shifted"` or `method="central"` in `power.TOST()` or `sampleN.TOST()`.

Of course it is recommended to use `method="exact"` ☺. There is no reason beside testing or comparative purposes to use an approximation if the exact method is available for no extra costs.

Both approximations can yield power values <0. In that case the power will be set =0.
To use these internal functions by yourself, you must supply the values diffm=log(Θ_0), theta1= log(Θ_1) and theta2= log(Θ_2) in case of log-transformed evaluation.
n is the sample size, df the degrees of freedom, bk the design constant.
It is highly recommended to use the high level functions `power.TOST()` or `sampleN.TOST()`. They shield you from all the peculiarities of the designs and log-transformed or un-transformed evaluation.

If you are interested in more insight in the implementation load down the source code tarball of the package *PowerTOST* from CRAN and have a look at the code and especially at the comments within it.
Do not hesitate to contact the maintainer in case of any question, feature request or observation of bug(s).

know.designs()

The function `known.designs()` contains all parameters specifically to use in the described formulas. Below is the output:

	no	design	df	df2	steps	bk	bknif	bkni	name
1	0	parallel	n-2	n-2	2	4.0	1/1	1.00000	2 parallel groups
2	1	2x2	n-2	n-2	2	2.0	1/2	0.50000	2x2 crossover
3	1	2x2x2	n-2	n-2	2	2.0	1/2	0.50000	2x2x2 crossover
4	2	3x3	2*n-4	n-3	3	2.0	2/9	0.22222	3x3 crossover
5	3	3x6x3	2*n-4	n-6	6	2.0	1/18	0.05556	3x6x3 crossover
6	4	4x4	3*n-6	n-4	4	2.0	1/8	0.12500	4x4 crossover
7	5	2x2x3	2*n-3	n-2	2	1.5	3/8	0.37500	2x2x3 replicate crossover
8	6	2x2x4	3*n-4	n-2	2	1.0	1/4	0.25000	2x2x4 replicate crossover
9	7	2x4x4	3*n-4	n-4	4	1.0	1/16	0.06250	2x4x4 replicate crossover
10	9	2x3x3	2*n-3	n-3	3	1.5	1/6	0.16667	partial replicate (2x3x3)
11	10	2x4x2	n-2	n-2	4	8.0	1/2	0.50000	Balaam's (2x4x2)
12	11	2x2x2r	3*n-2	n-2	2	1.0	1/4	0.25000	Liu's 2x2x2 repeated x-over
13	100	paired	n-1	n-1	1	2.0	2/1	2.00000	paired means

The bk are the 'design' constants in terms of n_{total} for balanced (sequence) groups, the bkni the 'design' constants in terms of the number of subjects (possibly unbalanced) in the (sequence) groups.

The df are the usual degrees of freedom, df2 the degrees of freedom for the so-called robust analysis, i.e. analysis via intra-subject contrasts T-R of the (log-transformed) values of the PK metrics. The df2 are also more appropriate if the planning of sample size is done based on CV's originating from real mixed model analysis (via Proc MIXED in SAS or lme() in R).

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