Local FDR Simulation Example

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This simulation example involves 2000 "genes", each of which has yielded a test statistic z_i, with $z_i \approx N(\mu_i, 1)$, independently for $i = 1, 2, \ldots, 2000$.

Here μ_i is the "true score" of gene i, which we observe only noisily. 1800 (90%) of the μ_i values are zero; the remaining 200 (10%) are from a $N(3, 1)$ distribution. The data are contained in the dataset `lfdrsim`, where the z_i are the column `zex`.

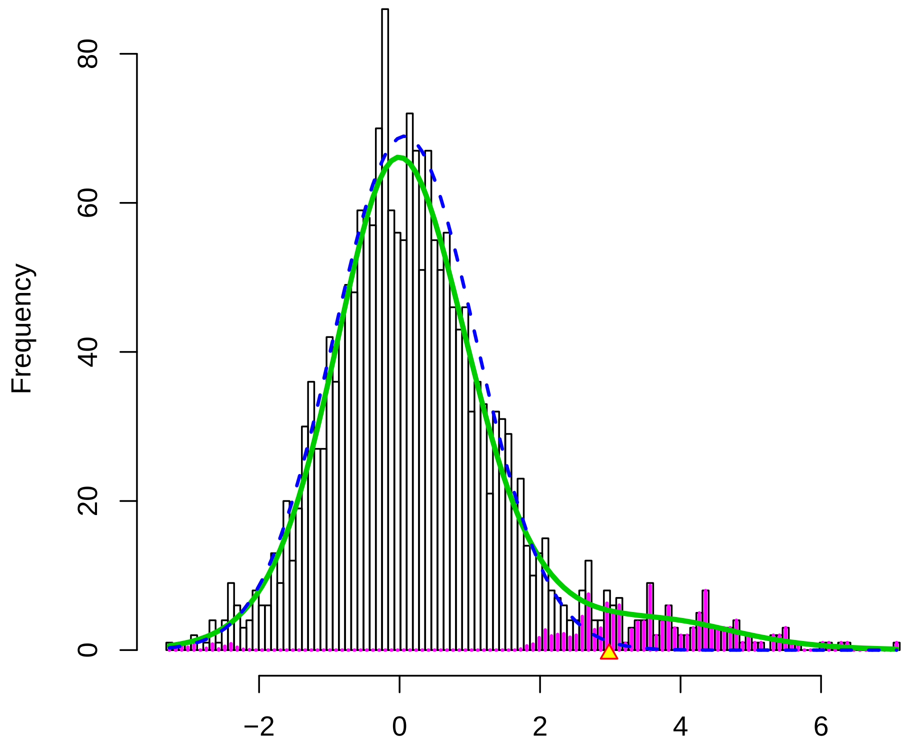
```
> library(locfdr)
```

```
Loading required package: splines
```

```
> data(lfdrsim)
> zex <- lfdrsim[, 2]
```

If we are confident that the null z_i's are distributed as $N(0, 1)$, we run `locfdr` with `nulltype=0`. Otherwise, we use the default `nulltype=1`, which uses empirical estimates of the null density parameters.

```
> w <- locfdr(zex)
```



MLE: delta: 0.071 sigma: 1.016 p0: 0.933
CME: delta: 0.011 sigma: 0.966 p0: 0.908

In the figure, the green solid line is the spline-based estimate of the mixture density f. The blue dashed line is the empirical null subdensity $p_0 f_0$, estimated by default by maximum likelihood (nulltype=1). Whichever nulltype is specified, `locfdr` returns a matrix `fp0` containing parameters of all three nulltypes and corresponding estimates of the proportion p_0 of cases that are null, along with standard errors. In this example, the null distribution is $N(0, 1)$, and both the MLE and central matching estimates come close to this.

```
> w$fp0

           delta      sigma          p0
thest 0.00000000 1.00000000 0.934884830
theSD 0.00000000 0.00000000 0.016381300
mlest 0.07133733 1.01567574 0.932555728
mleSD 0.02761442 0.02721782 0.009518058
cmest 0.01137651 0.96576676 0.908318708
cmeSD 0.04211370 0.03380724 0.013813796
```

The function `locfdr` returns, in the output `mat`, the bin centers `x`, and, at each `x`, the following values:

fdr local false discovery rate based on the specified nulltype

Fdrleft, Fdrright tail false discovery rates

f the mixture density estimate calculated using the type and df arguments, scaled to sum to the number of z_i's.

f0 the null density estimate calculated using the nulltype argument (using nulltype=1 if nulltype=0 is specified)

f0theo the null density estimate calculated using the theoretical null $N(0,1)$

fdrtheo the local false discovery rate based on the theoretical null $N(0,1)$

counts the number of z_i's in the bin

lfdrse the delta-method estimate of the standard error of the log of the local false discovery rate for the specified nulltype

p1f1 the estimated subdensity of the non-null z_i's

```
> w$mat[1:5, ]
```

```
             x       fdr   Fdrleft  Fdrright         f        f0    f0theo
[1,] -3.277130 0.4754348 0.4754348 0.9325557 0.5902186 0.3009048 0.3260307
[2,] -3.189391 0.5222393 0.5010207 0.9326907 0.7117024 0.3985595 0.4329734
[3,] -3.101651 0.5695273 0.5282337 0.9328368 0.8579789 0.5239820 0.5705853
[4,] -3.013912 0.6167842 0.5568976 0.9329928 1.0338087 0.6837521 0.7461681
[5,] -2.926172 0.6634879 0.5867905 0.9331566 1.2447492 0.8856050 0.9682989
       fdrtheo counts    lfdrse      p1f1
[1,] 0.5164208      1 0.3988950 0.3096081
[2,] 0.5687493      0 0.3698064 0.3400234
[3,] 0.6217304      1 0.3411065 0.3693365
[4,] 0.6747682      1 0.3129513 0.3961718
[5,] 0.7272533      2 0.2855029 0.4188731
```

The `fdr` in the result contains the local false discovery rate for each z_i. One might use this vector to create a list of Interesting cases.

```
> which(w$fdr < 0.2)
```

```
 [1]    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15
[16]   16   17   18   19   20   21   23   24   25   26   27   28   29   30   31
[31]   32   33   35   37   38   39   41   42   43   45   46   47   48   49   51
[46]   52   54   56   57   58   59   60   61   62   63   66   67   69   70   71
[61]   73   74   75   77   78   79   83   85   88   89   90   92   95   96   98
[76]  100  103  104  106  107  109  112  113  118  121  122  125  127  128  132
[91]  133  135  136  137  141  151  160  161  162  165  168  170 1732 1898
```

Here 0.2 is a rule-of-thumb cut-off. In the simulated data, the first 200 cases have nonzero μ_i. So we can find the true tail FDR.

```
> sum(which(w$fdr < 0.2) > 200)/sum(w$fdr < 0.2)
```

```
[1] 0.01923077
```

The estimated tail FDR can be found from the `mat` output.

```
> w$mat[which(w$mat[, "fdr"] < 0.2)[1], "Fdrright"]
```

```
[1] 0.03515483
```

The tail FDR is the mean local fdr over the entire tail and is therefore smaller than the local fdr cutoff.