GPoM : General introduction

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Generalized Global Polynomial Modelling (GPoM)

GPoM is an R package dedicated to the global modelling technique. It has been developped at the *Centre d'Etudes Spatiales de la Biosphère*[1, 2]. The global modelling technique is a model-building approach. Its aim is to obtain differential equations from time series. Model-building from data takes its background from the fields of Electrical Engineering and Statistics and was originally mostly dedicated to linear problems [3]. The more advanced developments of the model-building approaches have incorporated the Theory of Nonlinear Dynamical Systems in its background. Thanks to it, global modelling technique has become well adapted to model deterministic behaviours of various degree of nonlinarity (linear, weakly or strongly nonlinear, chaotic), and it is also well designed to model dynamical behaviors characterized by a high sentivity to the initial conditions.

In its Ordinary Differential Equations (ODEs) formulation, the global modelling technique was initiated in the early 1990s[4]. Its first illustrations were obtained thanks to a formalism developped by G. Gouesbet and his colleagues[5, 6]. The ability to obtain equations of a chosen system may highly vary depending on what variables are used to reconstruct the equations. The set of observed variables plays a very important role when trying to retrieve governing equations for any dynamical system. This question was investigated during the last decades[7, 8, 9].

It is only in the 2000s that a set of ODEs could be directly obtained from real world data set[10]. New algorithms were developped at the begining of the 2010s[11] that have proven to have a very high level of performance to model dynamical behaviors observed under real environmental conditions: cereal crops cycles, snow area cycles, eco-epidemiology, etc.[12, 13], etc.

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[2] The developments of the GPoM package have been funded by the french programs *LEFE / Insu* (2012-2016), and by the *Défi InFiNiTi / CNRS* (2016-2017)

[3] L. A. Aguirre & C. Letellier, Modeling nonlinear dynamics and chaos: A review, *Mathematical Problems in Engineering*, 2009, 238960.

[4] J. P. Crutchfield & B. S. McNamara, 1987. Equations of motion from a data series, *Complex Systems* **1**, 417-452.

[5] G. Gouesbet & J. Maquet, 1992. Construction of phenomenological models from numerical scalar time series, *Physica D*, **58**, 202-215.

[6] G. Gouesbet & C. Letellier, 1994. Global vector-field reconstruction by using a multivariate polynomial L2 approximation on nets, *Physical Review E*, **49**, 4955-4972.

[7] C. Letellier, L. A. Aguirre & J. Maquet, 2005. Relation between observability and differential embeddings for nonlinear dynamics, *Physical Review E*, **71**, 066213.

[8] L.A. Aguirre & C. Letellier, 2011. Investigating observability properties from data in non-linear dynamics. *Physical Review E*, **83**, 066209.

[9] L.A. Aguirre, L. L. Portes & C. Letellier, 2018. Structural, Dynamical and Symbolic Observability: From Dynamical Systems to Networks. *IEEE Transactions on Control of Network Systems*, arXiv:1806.08909v1.

[10] J. Maquet, C. Letellier & L. A. Aguirre 2007. Global models from the Canadian Lynx cycles as a first evidence for chaos in real ecosystems, *Journal of Mathematical Biology*, **55**(1), 21-39.

[11] S. Mangiarotti, R. Coudret, L. Drapeau & L. Jarlan, 2012. Polynomial search and global modeling: Two algorithms for modeling chaos," *Physical Review E*, **86**(4), 046205.

[12] S. Mangiarotti, L. Drapeau & C. Letellier, 2014. Two chaotic global models for cereal crops cycles observed from satellite in Northern Morocco, *Chaos*, **24**, 023130.

[13] S. Mangiarotti, Modélisation globale et caractérisation topologique de dynamiques environnementales: de l'analyse des enveloppes fluides et du couvert de surface de la Terre à la caractérisation topolodynamique du chaos, Habilitation to Direct

All these developments were initialy developped to model dynamical behaviors from single time series. Recent developments have shown that the global modelling technique can also be applied to model multivariate couplings[14, 15].

The present package provides global modelling tools for the modelling of linear and nonlinear behaviors directly from time series.

Seven illustrative `vignettes` are provided to introduce the package which can be used as a tutorial and as a demonstrator. These are as follows:

(1) `1 Conventions` introduces the conventions used to formulate sets of ODEs of polynomial form with `GPoM` and shows how to integrate them numerically,

(2) `2 PreProcessing` provides some simple examples of time series preprocessing before applying the global modelling technique,

(3) `3 Modelling` is dedicated to the global modelling itself. Several case studies are presented considering single and multiple time series, both for modelling or detecting causal couplings,

(4) `4 VisuOutput` shows how to get an overview of the output obtained with global modelling functions and explains how these are organised,

(5) `5 Predictability` provides basic examples of validation considering the models performances in term of predictability,

(6) `6 Robustness` illustrates the robustness of the global modelling technique under various types of degraded conditions: noisy time series, subsampling/resampling, short time series length, sensitivity to initial conditions,

(7) `7 Retromodelling` shows the ability of the global modelling technique to unveil the original equations when all the system variables are available.

The present GPoM package is made available to whom would like to use it. It includes most of the latest developments presently available for global modelling in ODE form, and we are happy to share it with you. Please refer to the following publications when using the present tools.

For univariate time series modelling: [1] S. Mangiarotti, R. Coudret, L. Drapeau & L. Jarlan, 2012. Polynomial search and global modeling: Two algorithms for modeling chaos," *Physical Review E*, **86**(4), 046205. https://journals.aps.org/pre/abstract/10.1103/PhysRevE.86.046205

For infering causal couplings and for detecting or analysing multivariate couplings: [2] S. Mangiarotti, 2015. Low dimensional chaotic models for the plague epidemic in Bombay, *Chaos, Solitons & Fractals*, **81**(A), 184-196. https://www.sciencedirect.com/science/article/pii/S0960077915002933

For using the generalized formulation of global modelling (that combines multiariate time series and some of their derivatives): [3] S. Mangiarotti, M. Peyre & M. Huc, 2016. A chaotic model for the epidemic of Ebola virus disease in West Africa (2013–2016). *Chaos*, **26**, 113112. https://aip.scitation.org/doi/abs/10.1063/1.4967730

For the time series resampling (before applying the global modelling technique): [4] S. Mangiarotti, 2018. The global modelling classification technique applied to the detection of chaotic attractors. https://ars.els-cdn.com/content/image/1-s2.0-S0960077917305040-mmc1.pdf *Supplementary Material A* to "Can the global modelling technique be used for crop classification?" by S. Mangiarotti, A.K. Sharma, S. Corgne, L. Hubert-Moy, L. Ruiz, M. Sekhar, Y. Kerr, 2018. *Chaos, Solitons & Fractals*, **106**, 363-378. https://www.sciencedirect.com/science/article/pii/S0960077917305040

Researches, Université de Toulouse 3, 2014.

[14] S. Mangiarotti, 2015. Low dimensional chaotic models for the plague epidemic in Bombay, *Chaos, Solitons & Fractals*, **81**(A), 184-196.

[15] S. Mangiarotti, M. Peyre & M. Huc, 2016. A chaotic model for the epidemic of Ebola virus disease in West Africa (2013–2016). *Chaos*, **26**, 113112.

For modelling the dynamics of aggregated (or associated) time series: [5] S. Mangiarotti, F. Le Jean, M. Huc, C. Letellier, 2016. Global modeling of aggregated and associated chaotic dynamics. *Chaos Solitons Fractals*, **83**, 82–96.

When topological properties can not be derived from the observational data and from the model (either due to noisy conditions, or high dimensional dynamic), alternative approaches have to be used for validation. Note that various validation methods have been introduced in [3]. https://aip.scitation.org/doi/abs/10.1063/1.4967730 Note that when a validation based on topological properties is possible, a validation of high precision can be performed as examplified in the supplementary matials https://journals.aps.org/pre/supplemental/10.1103/PhysRevE.86.046205/SuppMatos_PhysRevE_Pub_sept2012.pdf of reference [1] https://journals.aps.org/pre/supplemental/10.1103/PhysRevE.86.046205.

The authors of the package decline any responsability about the results and interpretations obtained and made by other users.